Integrating advanced discrete choice models in mixed integer linear optimization

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Outline

1. Introduction
2. General framework
3. Case study
4. Conclusions

1 Introduction

2 General framework

3 Case study

4 Conclusions

Motivation

Demand

- Choices of customers
- Discrete choice models
- Nonlinear and nonconvex formulations

Supply

- Design and configuration of the system
- Mixed Integer Linear Problems (MILP)

Demand model

- Population of N customers (n)
- Choice set $\mathcal{C}$ (i)
- $\mathcal{C}_n \subseteq \mathcal{C}$: alternatives considered by customer n

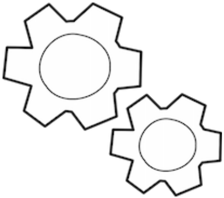
Behavioral assumption

- $U_{in} = V_{in} + \varepsilon_{in}$
- $V_{in} = \sum_k \beta_{ink} x^e_{ink} + q^d(x^d)$
- $P_n(i|\mathcal{C}_n) = \Pr(U_{in} \geq U_{jn}, \forall j \in \mathcal{C}_n)$

Simulation

- Distribution ε_{in}
- R draws $\xi_{in1}, \ldots, \xi_{inR}$
- $U_{inr} = V_{in} + \xi_{inr}$

Supply model



- Operator selling services to a market
  - Price p_{in} (to be decided)
  - Capacity c_i
- Benefit (revenue − cost) to be maximized
- Opt-out option ($i = 0$)

Price characterization

- Lower and upper bound
- Discretization: price levels
- Binary representation ($\lambda_{in\ell}$)

Capacity allocation

- Exogenous priority list of customers
- Here it is assumed as given
- Capacity as decision variable

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MILP (in words)

MILP

max	benefit
subject to	utility definition
	availability
	discounted utility
	choice
	capacity allocation
	price selection

Variables

Availability

$y_i \in \{0,1\}$	services proposed by the operator
$y_{in} \in \{0,1\}$	$y_i = 1$ and services considered by customers
$y_{inr} \in \{0,1\}$	capacity restrictions

Utility and choice

U_{inr}	utility
z_{inr}	discounted utility
U_{nr}	maximum discounted utility
$w_{inr} \in \{0,1\}$	choice

Pricing

$\lambda_{in\ell} \in \{0,1\}$	binary representation of the price
$\alpha_{inr\ell} \in \{0,1\}$	linearization of the product $w_{inr}\lambda_{in\ell}$

MILP

MILP

max	benefit
subject to	**utility definition**
	availability
	discounted utility
	choice
	capacity allocation
	price selection

Utility

$$U_{inr} = \overbrace{\beta_{in} p_{in} + q_d(x_d)}^{V_{in}} + \xi_{inr} \quad \forall i, n, r \quad (1)$$

p_{in}	endogenous variable
β_{in}	associated parameter ($\beta_{0n} = 0$)
$q_d(x_d)$	exogenous demand variables

MILP

MILP

max	benefit
subject to	utility definition
	availability
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$$y_{in}^d = \begin{cases} 1 & \text{if } i \in \mathcal{C}_n \\ 0 & \text{otherwise} \end{cases} \quad \forall i, n$$

Product of decisions

$$y_{in} = y_{in}^d y_i \quad \forall i, n \tag{2}$$

Availability at operator and scenario level

$$y_{inr} \leq y_{in} \quad \forall i, n, r \tag{3}$$

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$$z_{inr} = \begin{cases} U_{inr} & \text{if } y_{inr} = 1 \\ \ell_{nr} & \text{if } y_{inr} = 0 \end{cases} \qquad \forall i, n, r$$

(ℓ_{nr} smallest lower bound)

Discounted utility

$$\ell_{nr} \leq z_{inr} \qquad \forall i, n, r \quad (4)$$

$$z_{inr} \leq \ell_{nr} + M_{inr} y_{inr} \qquad \forall i, n, r \quad (5)$$

$$U_{inr} - M_{inr}(1 - y_{inr}) \leq z_{inr} \qquad \forall i, n, r \quad (6)$$

$$z_{inr} \leq U_{inr} \qquad \forall i, n, r \quad (7)$$

MILP

MILP

max	benefit
subject to	utility definition
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$$U_{nr} = \max_{i \in \mathcal{C}} z_{inr} \qquad \forall n, r$$

$$w_{inr} = \begin{cases} 1 & \text{if } i = \arg\max\{U_{nr}\} \\ 0 & \text{otherwise} \end{cases} \qquad \forall i, n, r$$

Choice

$$z_{inr} \leq U_{nr} \qquad \forall i, n, r \qquad (8)$$

$$U_{nr} \leq z_{inr} + M_{nr}(1 - w_{inr}) \qquad \forall i, n, r \qquad (9)$$

$$\sum_i w_{inr} = 1 \qquad \forall n, r \qquad (10)$$

$$w_{inr} \leq y_{inr} \qquad \forall i, n, r \qquad (11)$$

MILP

MILP

max	benefit
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Capacity allocation

- Priority list
- Two sets of constraints $\forall i > 0$
  - Capacity cannot be exceeded ($\Rightarrow y_{inr} = 1$)
  - Capacity has been reached ($\Rightarrow y_{inr} = 0$)

Price selection

$$p_{in} = \frac{1}{10^k}\left(\ell_{in} + \sum_{\ell=0}^{L_{in}-1} 2^{\ell}\lambda_{in\ell}\right)$$

- When calculating the benefit: $\lambda_{in\ell} w_{inr}$
- $\alpha_{inr\ell} = \lambda_{in\ell} w_{inr}$ + linearizing constraints

MILP

MILP

max	**benefit**
subject to	utility definition
	availability
	discounted utility
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	capacity allocation
	price selection

$$\max \sum_{i>0} (R_i - C_i)$$

Revenue

$$R_i = \frac{1}{R}\frac{1}{10^k}\left[\sum_n \sum_r \left(\ell_{in} w_{inr} + \sum_\ell 2^\ell \alpha_{inr\ell}\right)\right]$$

Cost

$$C_i = (f_i + v_i c_i) y_i$$

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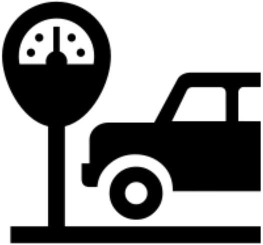
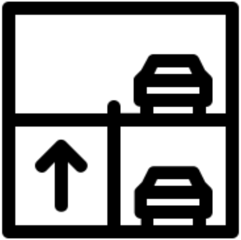
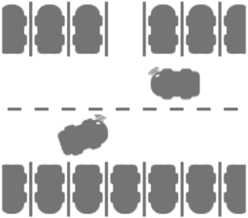
Parking choices

- $N = 50$ customers
- $\mathcal{C} = \{\text{PSP}, \text{PUP}, \text{FSP}\}$
- $\mathcal{C}_n = \mathcal{C} \quad \forall n$
- PSP: $0.50, 0.51, \ldots, 0.65$ (16 price levels)
- PUP: $0.70, 0.71, \ldots, 0.85$ (16 price levels)
- Capacity of 20 spots

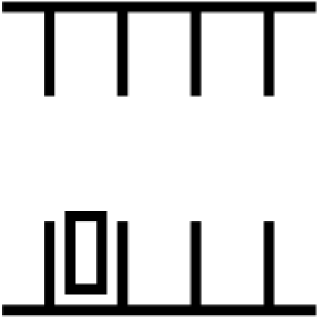
Choice model: mixtures of logit model[1]

$$
\begin{aligned}
V_{FSP} &= \bigcirc\!\!\!\beta_{AT}\, AT_{FSP} + \boxed{\beta_{TD}}\, TD_{FSP} + \boxed{\beta_{Origin_{INT\_FSP}}}\, Origin_{INT\_FSP} \\
V_{PSP} &= \boxed{ASC_{PSP}} + \bigcirc\!\!\!\beta_{AT}\, AT_{PSP} + \boxed{\beta_{TD}}\, TD_{PSP} + \bigcirc\!\!\!\beta_{FEE}\, \mathbf{FEE_{PSP}} \\
&\quad + \boxed{\beta_{FEE_{PSP(LowInc)}}}\, \mathbf{FEE_{PSP}}\, LowInc + \boxed{\beta_{FEE_{PSP(Res)}}}\, \mathbf{FEE_{PSP}}\, Res \\
V_{PUP} &= \boxed{ASC_{PUP}} + \bigcirc\!\!\!\beta_{AT}\, AT_{PUP} + \boxed{\beta_{TD}}\, TD_{PUP} + \bigcirc\!\!\!\beta_{FEE}\, \mathbf{FEE_{PUP}} \\
&\quad + \boxed{\beta_{FEE_{PUP(LowInc)}}}\, \mathbf{FEE_{PUP}}\, LowInc + \boxed{\beta_{FEE_{PUP(Res)}}}\, \mathbf{FEE_{PUP}}\, Res \\
&\quad + \boxed{\beta_{AgeVeh_{\leq 3}}}\, AgeVeh_{\leq 3}
\end{aligned}
$$

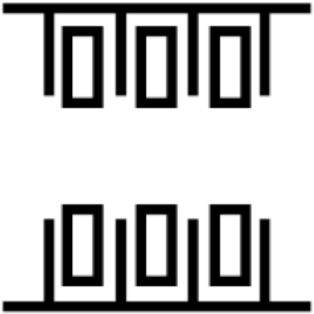
- Parameters
  - Circle: distributed parameters
  - Rectangle: constant parameters
- Variables: all given but FEE (in bold)

[1] A. Ibeas, L. dellOlio, M. Bordagaray, et al., "Modelling parking choices considering user heterogeneity," Transportation Research Part A: Policy and Practice, vol. 70, pp. 41 –49, 2014.

Experiment 1: uncapacitated vs capacitated case (1)



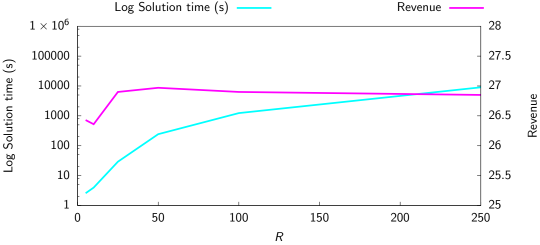
- Capacity constraints are ignored
- Unlimited capacity is assumed



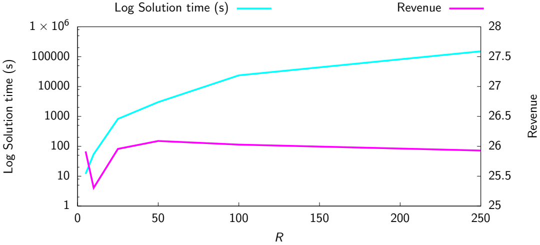
- 20 spots for PSP and PUP
- Opt-out has unlimited capacity

Experiment 1: uncapacitated vs capacitated case (2)

Uncapacitated

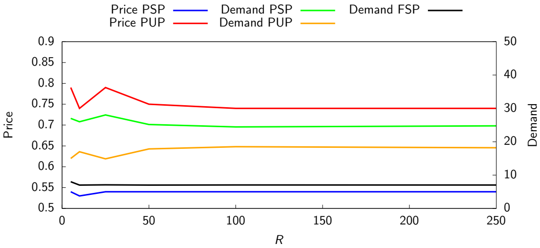


Capacitated

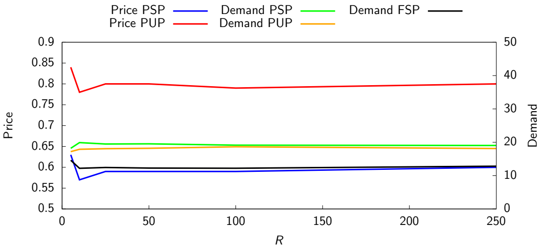


Experiment 1: uncapacitated vs capacitated case (3)

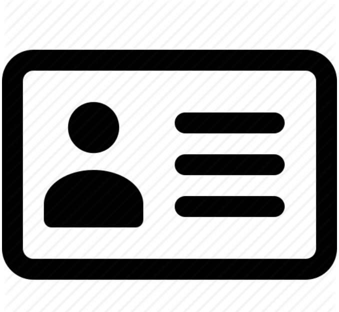
Uncapacitated

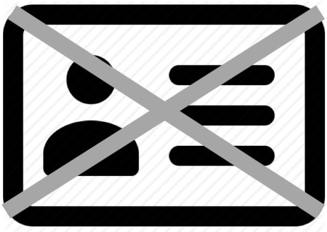


Capacitated



Experiment 2: price differentiation by segmentation (1)

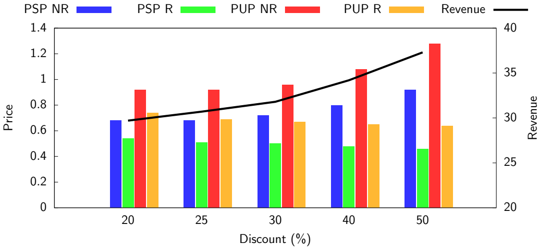




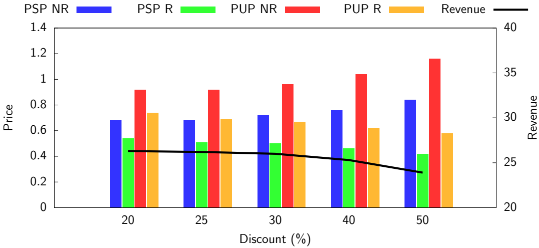
- Discount offered to residents
- Two scenarios (municipality)
  1. Subsidy offered by the municipality
  2. Operator obliged to offer reduced fees
- We expect the price to increase
  - PSP: $\{0.60, 0.64, \ldots, 1.20\}$
  - PUP: $\{0.80, 0.84, \ldots, 1.40\}$

Experiment 2: price differentiation by segmentation (2)

Scenario 1

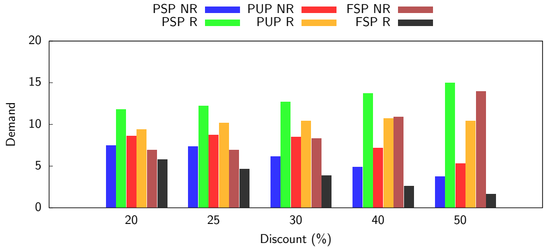


Scenario 2

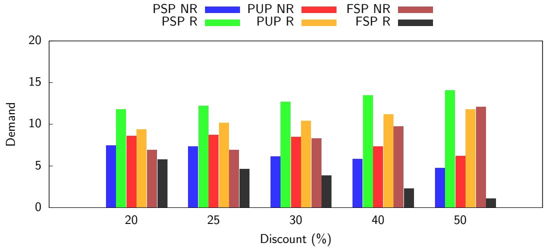


Experiment 2: price differentiation by segmentation (3)

Scenario 1



Scenario 2



Other experiments

Impact of the priority list

- Priority list = order of the individuals in the data (i.e., random arrival)
- 100 different priority lists
- Aggregate indicators remain stable across random priority lists

Benefit maximization through capacity allocation

- 4 different capacity levels for both PSP and PUP: 5, 10, 15 and 20
- Optimal solution: PSP with 20 spots and PUP is not offered
- Both services have to be offered: PSP with 15 and PUP with 5

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Conclusions and ongoing research

Conclusions

- Powerful tool to configure systems based on heterogenous behavior
- Computationally expensive, e.g., for $N = 50$ and $R = 250$
  - Uncapacitated: 2.5 h
  - Capacitated: 1.7 days
- In practice, more individuals and a high number of draws is desirable

Ongoing research

- Decomposition technique (Lagrangian relaxation)
- Faster subproblems that can be parallelized

Questions?